



AHP ALGORITHM IN DATA DIFFUSION ACROSS A SOCIAL NETWORKING PLATFORM¹

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Abstract

The Social Networking platforms (SNPs) have become major media of communication, information dissemination and sharing. SNP ignores the constraint of distance, time and type of information during communication among users. It is obviously seeming the best platform when it comes to marketing, advertisement and information transmission. In this paper, content ranking model was used to advance on how contents on a social networking platform can be shared among users faster and hereby improving the rate of information transmission over the network. Analytic Hierarchy Process (AHP) algorithm was deployed to find the level of importance in the constructed metadata.

Keywords: Social Networking platform, Communication, Information sharing, and Content Ranking Model.

Introduction

It is believed generally that information is life and its importance in humans' endeavours cannot be over emphasised. The fast spread of information in today's digital age is contributing largely to the world's current



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greatest achievements in preservation of data and its accessibility. People need to access relevant data across the globe for different reasons and purposes. The coming of social networking platforms have made rate of information dissemination easiest and at same time improving learnings in our present generation.

Information is any kind of event that affects the state of a dynamic system. This information could be represented and stored in different formats and mode like audio, video, plain text or transmitted signal. Liu (2016) asserts that users of social networks face a problem with data dissemination as the number of people using mobile devices like smart phones and tablets has significantly increased over time. Now that short-range radio stations are built into mobile devices, mobile users can easily access the content they're interested in. In contrast to routing protocols, which only focus on determining the best route between two ends of a network, data diffusion imports packets from multiple sources to numerous endpoints.

Recognising social network as one of the major platforms that has aided in the spread of data across the globe by the means of providing an environment where people get to interact and share ideas with one another. Companies, government and individuals have also been able to get relevant information across to their desired audience through advertisement provided by the social networking bodies. case of an online petition. They argued that people regularly use means of online social networks. Since social networking sites like Facebook, Google, WhatsApp, and Twitter first appeared on the scene, an unprecedented number of individuals have been using them as a medium for communication and the sharing of thoughts. Nyagadza (2020) asserts that businesses are more likely to use social media to advertise their products to target audiences or customers because of its viral potential. The study on search engine marketing and social media marketing predictive trends demonstrates that an intuitive understanding of the digital marketing predictive trends is essential for creating an agile stance to outperform competitors in blue oceans. Due to its accessibility, effectiveness, and substantial broadcasting ability for information sharing, Mohammad etaal (2016) researched the spread of information through social networks. They believed the spread of petitions in social networks raises various theoretical and practical questions. They deployed system dynamics modelling to address these questions and petitions diffusion online.

Lots of users had sign up on these platforms for interaction purpose. The variety of users and content available on these social platforms are what makes the activities on these platforms exciting. Data sharing among users on these networks is growing daily due to accessibility and availability of relevant contents that matches the



interests of these users, makes social networks essential and dependable among populace of nations. Kawaljeet etaal (2018) came to the conclusion that they were able to identify multiple emergent themes in existing corpus, which furthered their understanding of advances in social media research and it can prevent duplication by future researchers. They came up with fruitful conclusions after examining 132 papers (in selected IS journals) on social media and social networking published between 1997 and 2017. The study was titled advances in social media research: past, present, and future.

Due to the ease with which these tasks may now be completed with a few button clicks, the accessibility level for collaboration and global communication has significantly increased. A social network is essentially an online community where members may build public profiles, communicate with friends and other users in real time, and connect with others who share their interests. Written letters sent by hand from one person to another was one of the first means of communication over long distances. In any case, throughout time, this has altered. Lai etaal (2020) claim that COVID-19 has highlighted a chance for medical professionals to participate in online Public Engagement with Science (PES) via Reddit as a social network platform that offers an Ask Me Anything (AMA) format for subject matter and enables experts to respond to questions posed by users (public). Based on the analysis of the questions and answers they conducted on an AMA organised by Massachusetts General Hospital in March 2020 to provide emergency medical care to the public by hospital physicians, they found that participants (public) sought not only to obtain information but also to engage in discussion and did so with each other in the absence of expert responses. They came to the conclusion that while most of the dialogue was reciprocal and involved participants, not all questions could be fully addressed due to the volume of queries and the expert-to-participant ratio. This implies that postings offering information or ideas were more common than posts requesting or offering resources.

At the individual, organisational, and community levels, the social impact of social media can be assessed. Businesses take advantage of the multi-level construct by maximising their capacity to influence people's attitudes, knowledge, and beliefs through social media. This in turn influences the environment at the local level as well as the organisational structure of the mass media. Social media has evolved from merely serving as a means for people to stay in touch with their loved ones to becoming online hubs where customers can research and learn more about the products and services that interest them, the businesses that make them, and where to buy and have them delivered to their doorsteps without ever leaving their homes. These websites are being used by marketers and merchants to reach



consumers in new ways and offer them new ways to shop. Technology-related advancements including the emergence of potent search engines, cutting-edge mobile devices and interfaces, peer-to-peer communication tools, and online social networks have increased marketers' capacity to contact consumers through new touch points (Shankar et al., 2011).

A social network can be basically seen as a structure of ties or relational variables, which exist between social actors, nonetheless of the number. It's liken to a fixed set, which by the matrices could be represented by $\{1, 2, \dots, n\}$ set of actors, and variables X_{ij} , which is simply a notation that represents how actor i is tied to actor j . If the set $\{1, 2, \dots, n\}$ is taken to be the number of actors in a model, then the mathematical object constituted by the set and the variables X_{ij} is called a graph positioned in the non-directed and a diagram in the directed case. Data structures designed for binary variables are the ones that are most commonly used today, and by using this model, it is possible to say that the social network depends on concepts like reciprocation, homophily, transitivity, degree differentials, and hierarchies that have both empirical and theoretical support. Homophiles are simply the tendency of similar players in a network to relate to one another readily, and it is said that Lazarsfeld and Merton first identified this phenomenon in 1954. Theoretical justifications for this include chance, affinity, ease of communication, reduced transaction costs, and lower break-off risks, and it is thought to have promoted stronger bonds between actors with like-minded beliefs (Snijders, 2011). Juan etaal (2018) discover that elements of the Elaboration Likelihood Model (ELM) on both the core and peripheral routes have favourable effects on the individual dissemination behaviour of social networking site users. The factors that are related to relationships and information receiver have the most impact. Contrarily, source-related factors have the least impact. They came to the conclusion that the effect of value homophily on a person's dissemination behaviour is almost entirely (50%) mediated by the level of social ties..

Lyu and Lai's (2022a) work looked at how second language (L2) learners interact with peer criticism in educational settings. It is mainly unknown how these students interact with online communities outside of the classroom in self-initiated and self-directed feedback activities. They investigated in-depth the dynamics and underlying processes of learners' engagement with written peer feedback on a social networking site for L2 writers while taking an ecological perspective into consideration. According to their research, fostering socially relevant feedback, improving specific platform capabilities,



cultivating a sense of community, and assisting learners in rethinking the platform in light of their language learning are essential components of improving learners' engagement with peer feedback. These also point to the necessity of a flexible and comprehensive strategy for comprehending and fostering students' interaction with feedback outside of the classroom.

In order to improve second language (L2) learners outside of the classroom, Lyu and Lai (2022b) claim that encouraging learner engagement on technological platforms in an informal learning setting and offering insights into how such assistance could be done through system design are crucial.

Toivonen (2009) provided a model for social networks that incorporates common assumptions about how such networks should collect, organise, and make available data. In that piece, he explains how social networks are structured into communities with many interconnected nodes, which leads to large clustering coefficients. Toivonen's model is driven by an algorithm that incorporates both random and implicit attachments as growth processes. It was claimed that the degree of dispersion is determined by the amount of edges formed by the second process for each random attachment, and that the implicit preferred attachment process gives rise to highly clustered, assortative, and community-based structures.

Wenlin et al. (2018), in their article "Tweeting about emergency: Semantic network analysis of government agency social media posts during Hurricane Harvey revealed three main takeaways: (a) crisis response strategies are ranked differently across multiple crisis stages; (b) Twitter improves government agency-public crisis communication; and (c) government agency crisis communication strategies prioritise information over blame mitigation.

Statement of the Problem

Social network ostensibly is of now about the major platform that has facilitated in the spread of data across the globe by the means of providing an environment that encourages interaction and content sharing. The problem for the need of delivery of rich content that matches the interest of users which would in the end advances their intellectual, social and educational know how is hereby measured. It is believed that this study could help users of social network platforms to share better information, find relevant data for researches and aid the world in its information and communication technology development. The problem of data diffusion that could contribute in the enhancement of its flow across the social platforms by applying a mathematical model that could increase the rate at which accurate data is made available to the users based on their interest and



location is what we tried to solve in this paper.

Aim and Objective of Study

With this research, we hope to implement a mathematical model that will speed up the process by which users of a future interest- and location-based social network will have access to reliable information. The following aims were pursued in order to reach this goal:

- 1) A content ranking model was used to index every activity of a user on a social network, builds up these indexes with Meta data.
- 2) We deployed the Analytic Hierarchy Process (AHP) algorithm to determine the level of importance in the built metadata.

Significance of the Study

The delivery of suitable and correct information can help to solve almost of the world's problem and can serve as a development catalyst. No information should be undervalued seeing the fact that one of the greatest invention of all time (Gravity) was discovered as a result of a simple apple which dropped from its parent tree. Supposing a social graph that maps the relationship of users who carry out different activity on a network; there is a probability that the activity carried out by one user might be tremendously helpful or rather match the interest of another user. For instance, by means of the pre-existing models of social networks like Facebook, where users are only likely to access information that friends share or information that was gotten by self-effort; and the probability of the user who's interest matches the activity of the other will depend on either the user's self-effort to run a serious search for such information (provided none of the users in his friends circle shares or likes this information) or by accessing this information via the suggestion widget. Therefore, the significance of this study is derived from a model that could analyse a user's interest, build up a training set of data and uses it to predict the user's next possible activity.

Scope of Study

The scope of this study would be to enhance the accessibility of contents and also provides the basic primary mode of communication between users. It focuses on constructing a real time social application with a text limit per post (this limitation feature makes users on this network to be more précised when authoring post), seeing updates of users mapped to user's circle of friends, following/sharing/like topics or posts and building a list of activity/post suggestions that validated what a user might be interested in. The scope limited toward text-based information streaming which means that any other format of information representation like video and audio may deliberately not be attended to.

**Mathematical Background:**

Theorem 1: Let A and B be subsets of a set X . Then $A \subset B$ if and only if one of the following conditions holds:

$$\begin{aligned} A \cup B &= A, \\ B &= A \cup B, \\ X \sim B &\subset X \sim A, \\ A \cap X \sim B &= \emptyset, \text{ or} \\ (X \sim A) \cup B &= X. \end{aligned}$$

Theorem 2: Let A, B, C and X be sets.

Then:

- $X \sim (X \sim A) = A \cap X$.
- $A \cup B = B \cup A$ and $A \cap B = B \cap A$. - Commutative laws,
- $A \cup (B \cap C) = (A \cup B) \cap C$ and $A \cap (B \cup C) = (A \cap B) \cup C$. - Associative laws,
- $A \cup (B \cap C) = (A \cap B) \cup (A \cap C)$ and $A \cap (B \cup C) = (A \cup C) \cap (A \cup B)$. - Associative laws,
- $X \sim (A \cup B) = (X \sim A) \cap (X \sim B)$ and $X \sim (A \cap B) = (X \sim A) \cup (X \sim B)$. - DeMorgan Formula. (See Appendix A.1).

Theorem 3: Let A be an index set, and for each a in A let X_a be a subset of a fixed set Y . then:

- If B is a subset of A , then $\bigcup \{X_b : b \in B\} \supset \bigcup \{X_a : a \in A\}$ and $\bigcap \{X_b : b \in B\} \supset \bigcap \{X_a : a \in A\}$.
- (De Morgan formulae)
 $\bigcap \{X_a : a \in A\} = \bigcap \{Y \sim X_a : a \in A\}$ and $Y \sim \bigcap \{X_a : a \in A\} = \bigcup \{Y \sim X_a : a \in A\}$.
 (Kelley, 1688).

Relations

Suppose that we are given a relation (in the intuitive sense) between certain pairs of objects. The basic idea is that the relation may be represented as the set of all pairs of mutually related objects. The basic facts which we need here are: each ordered pair has a first coordinate and a second coordinate, and two ordered pairs are equal (identical) if and only if they have the same first coordinate and the same second coordinate. The ordered pair with first coordinate x and second coordinate y is denoted (x, y) . Thus $(x, y) = (u, v)$ if and only if $x = u$ and $y = v$.



A relation is a set of ordered pairs; that is, a relation is a set, each member of which is an ordered pair. If R is a relation we write xRy and $(x, y) \in R$ interchangeably, and we say that x is R -related to y if and only if xRy . The domain of a relation R is the set of all first coordinates of members of R , and its range is the set of all second coordinates.

Theorem 4: Let R , S , and T be relations and let A and B be sets. Then:

- a) $(R^{-1})^{-1} = R$ and $(R \bullet S)^{-1} = S^{-1} \bullet R^{-1}$
- b) $R \bullet (S \bullet T) = (R \bullet S) \bullet T$ and $(R \bullet S)[A] = R[S[A]]$
- c) $R[A \cup B] = R[A] \cup R[B]$ and $R[A \cap B] \subset R[A] \cap R[B]$

More generally, if there is given a set X_a for each member of a non-void index set A then:

d)

$R[\cup\{X_a : a \in A\}] = \cup\{R[X_a] : a \in A\}$ and $R[\cap\{X_a : a \in A\}] \subset \cap\{R[X_a] : a \in A\}$. (See Appendix A.2).

Theorem 5: A relation R is an equivalence relation if and only if there is a disjoint family \mathcal{a} such that $R = \cup\{A \times A : A \in \mathcal{a}\}$. (See Appendix A.3).

Functions

A function is a relation such that no two distinct members have the same first coordinate. Thus, f is a function if and only if the members of f are ordered pairs, and whenever (x, y) and (x, z) are members of f , then $y = z$. A function is a set, and consequently two functions, f and g , are identical if and only if they have the same members.

If A is a set and f is a function, then, following the definition given for arbitrary relations, $f[A] = \{y : \text{for some } x \text{ in } A, (x, y) \in f\}$; equivalently, $f[A]$ is $\{y : \text{for some } x \text{ in } A, y = f(x)\}$. The set $f[A]$ is called the image of A under f . If A and B are sets, then, by theorem 4, $f[A \cup B] = f[A] \cup f[B]$ and $f[A \cap B] \subset f[A] \cap f[B]$, and similar formulae hold for arbitrary unions and intersections. It is not true in general that $f[A \cap B] = f[A] \cap f[B]$, for disjoint sets may have intersecting images. If f is a function, then the set $f^{-1}[A]$ is called



the inverse (inverse image, counter image) of A under f . The inverse satisfies the following algebraic rules.

Theorem 7: If f is a function and A and B are sets then

$$a) f^{-1}[A \cap B] = f^{-1}[A] \cap f^{-1}[B],$$

$$b) f^{-1}[A \cup B] = f^{-1}[A] \cup f^{-1}[B].$$

More generally, if there is given a set X_c for each member c of a non-void index set C then

$$c) f^{-1}[\cup\{X_c : c \in C\}] = \cup\{f^{-1}[X_c] : c \in C\}, \text{ and}$$

$$d) f^{-1}[\cap\{X_c : c \in C\}] = \cap\{f^{-1}[X_c] : c \in C\},$$

(See Appendix A.4)

Content Indexing and Ranking

Given a set of users U that exist in a social networking platform, and a set of contents C that exists within the same platform.

$$C = \{c_1, c_2, c_3 \dots c_n\}$$

$$U = \{u_1, u_2, u_3 \dots u_n\}$$

For every content the user finds interesting, an index set is created and populated with previous contents that have high integrity from the set of contents the involved user likes. We denote the index set as $N = \{n_1, n_2, n_3 \dots n_i\}$ where $n_i \subset C$ and for every c_i there exists an index $n_i \in N$.

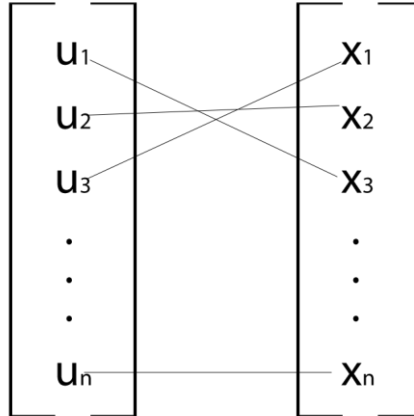
In general, it simply means that if a user likes the content c_1 and proceeds to liking content c_2 , a relation can be defined $c_1 R c_2$ which an element is of n_1 . This relation could necessarily not be transitive until another user gets to like c_1 from the index of c_2 i.e. n_2 .

Let $X = \{x_i : a_i \in x_i\}$ where $x_i \subset C$ this implies that for every $a \in x_i, a \in C$.

Taking x_i as a set of contents a user is interested in, we can assign a

value y to u_i where

$u_i \in U$ and $y = f(u_i)$. Below represents the mapping from U to X .



Each user in the map above is mapped to a well-defined set of contents of x_i . Further exploring into the set x_i which is the set of contents the user finds interesting. Say a_1 is an element of x_i with the index n_1 , if the mapped user finds any content in the index n_1 interesting, the integrity of a_1 increases by the constant K , where $k = 0.5$.

For every a_i there exists a mapping $z_i = f(z_i)$.

This implies that $f(z_i) = \sum_0^j 0.5(j)$ where j = number of contents in the index n_i .

If a random content (c_3) is liked by the user "A", the elements found in the index of c_3 which is n_3 will be ordered by the level of their importance using the A.H.P algorithm with the total numbers of index rank as its parameter. Suggesting a fixed number of elements from the ordered set to the user.

System Model

Assume a small social network for predicting user interest (list of suggestions retrieved based on the previous activities carried out by

the user), consisting of four contents Programming, Networking, Mathematics and Statistics. Let the table below be the indexed content of Networking. We can calculate the level of importance of these elements in the index and know what to suggest to the next user that decides to like Networking.

Programming	Mathematics	Statistics
Rating = 2	Rating = 3	Rating = 1
Location = Redeemers	Location = Covenant	Location = Redeemers

Table 4.1

Implementing the Analytic Hierarchy Process (AHP) in C++ programming language, we can find the level of importance of each of the content in the contained table above. Using the 1 – 9 scaling for quality data such as preference, ranking and subjective opinions. We use the provided details above to obtain the pair-wise

comparison, by simply apply the formula $a_{ij} = (x - y) / (x / 9)$. If

a_{ij} is negative, then the given result (a_{ij}) is multiplies by minus times

the square root of its reciprocal else if $x = y$, then $a_{ij} = 1$. Taking networking as a course in Covenant.

By Ranking, we have the table below

$$\begin{array}{c}
 a \quad b \quad c \\
 \left(\begin{array}{ccc}
 1 & \frac{1}{5} & 5 \\
 5 & 1 & 6 \\
 \frac{1}{5} & \frac{1}{6} & 1
 \end{array} \right) \\
 w_{ij} = w_{ji}^{-1} = \frac{1}{w_{ji}}
 \end{array}$$

$$AB = 2, BA = \frac{1}{2} \text{ that is the reciprocal of } AB$$

Steps

- i. Find the degree of importance of each requirement
- ii. Calculate consistency index (CI)
- iii. Calculate random index (RI)
- iv. Calculate the consistency ratio (CR) and determine if the judgment is acceptable or not

Formulae

$$CI = \frac{\lambda_{\max} - n}{n - 1}$$

$$RI = \frac{1.98(n - 2)}{n}$$

$$CR = \frac{CI}{RI}$$

Implementation of A.H.P Algorithm in c++

```
/* * File: AHP.h; Author: Constantine; Created on May 20, 2014, 7:04 PM; */
```

```
#include <vector>
#ifndef AHP_H
#define AHP_H
using namespace std;
typedef struct {
    double lamdaMax; double CI; double RI; double CR;
} resultSet;
class AHP {
public:
    AHP(int mysize);
    AHP(const AHP&orig);
    virtual ~AHP();
    void add(double *objectSet);
    vector<double *> dataSet;
    vector<double *> DOM;
```

```

vector<double *>degreeOfImportance();
double *getSumOfRoll();
double *normalize();
double *resultingVector();
resultSetgetResultSet();
private:
resultSetresultSet;
int size;
};
#endif /* AHP_H */
(See Appendix B.1)

```

Discussion of Result

Calculating the level of importance of the provided contents with the program written above, we got the following values;

Programming	Statistics	Mathematics
5	1.17647	5

Table 4.2

Judging from the result above, programming will be the first content to be suggested to anyone who likes networking irrespective of the location because it has a high ranking and also comes first in the index table. Mathematics will come second and lastly Statistics because it has the lowest importance level. The ranking of each content in the networking index will be rebuilt as users taste changes at run time.

Conclusion

Every research and project carried out today was aided by past works and discovery. Imagine we never had access to these data, what would have been our hope? Developing the user's learning curve and the reduction in amount of time and constraints that are needed to access these data could be reduced by the application of this model on a social networking platform. In this paper we had shown that we can possibly build up training sets of users' activities; index these sets and analytically tell what next a user could possibly be



interested in instead of retaining the transitivity relationship and keeping the suggestion box within the close relational map.

Recommendations

The suggested model is adaptable; therefore it is possible to add restrictions to increase the likelihood that the resulting solution would be accurate enough for practical prediction purposes. It is suggested that additional research be conducted in the area of updating the model to account for the weighting of materials already present in an index.

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Appendix

A.1 Proof of Theorem 2

Proof of (a): A point x is a member of $X \sim (X \sim A)$ if and only if $x \in X$ and $x \in X \sim A$. Since $x \in X \sim A$ if and only if (iff) $x \in X$ or $x \in A$, it follows that $x \in X \sim (X \sim A)$ if and only if $x \in X$ and either $x \in X$ or $x \in A$. The first of these alternatives is impossible, so that $x \in X \sim (X \sim A)$ iff $x \in X$ and $x \in A$; that is iff $x \in X \cap A$. Hence, $X \sim (X \sim A) = A \cap X$. Proof of first part of (d): A point x is

a member of $A \cap (B \cup C)$ iff $x \in A$ and either $x \in B$ or $x \in C$. This is the case iff either x belongs to both A and B or x belongs to both A and C . Hence $x \in A \cap (B \cup C)$ if and only if $x \in (A \cap B) \cup (A \cap C)$, and equality is proved.

If A_1, A_2, \dots, A_n are sets then $A_1 \cup A_2 \cup \dots \cup A_n$ is the union of the sets and $A_1 \cap A_2 \cap \dots \cap A_n$ is their intersection. It does not matter how the terms are grouped in computing the union or intersection because of the associative laws. We shall also have to consider the union of the members of non-finite families of sets and it is extremely convenient to have a notation for these unions. Consider the following situation: for each member a of set A , which we call an index set, we suppose that a set X_a is given. Then the union of all X_a denoted by $\bigcup \{X_a : a \in A\}$ is defined to be the set of all point x such that $x \in X_a$ for each a in A . A very important special case arises when the index set is itself a family a of sets and X_A is the set A for each A in a . Then the foregoing definition becomes:

$$\bigcup \{A : A \in a\} = \{x : x \in A \text{ for some } A \text{ in } a\} \text{ and } \bigcap$$

$$\{A : A \in a\} = \{x : x \in A \text{ for each } A \text{ in } a\}$$

A.2 Proof of Theorem 4

As an example we prove the quality: $(R \bullet S)^{-1} = S^{-1} \bullet R^{-1}$. A pair (z, x) is a member of $(R \bullet S)^{-1}$ iff $(x, z) \in R \bullet S$, and this is the case iff for some y it is true that $(x, y) \in S$ and $(y, z) \in R$. Consequently $(z, x) \in (R \bullet S)^{-1}$ iff $(z, y) \in R^{-1}$ and $(y, x) \in S^{-1}$ for some y . This is precisely the condition that (z, x) belong to $S^{-1} \bullet R^{-1}$.

There are several special sorts of relations which occur so frequently in mathematics that they have acquired names. Aside from orderings and functions, which will be considered in detail in the following sections, types listed below are probably the most useful. Throughout the following it will be convenient to suppose that R is a relation and that X is the set of all points which belong to either the domain or the range of R ; that is, $X = (\text{domain } R) \cup (\text{range } R)$. The relation R is reflexive if and only if each point of X is R -related to itself. This is entirely equivalent to requiring that the identity Δ (or $\Delta(X)$) be a subset of R . The relation R is symmetric, provided that xRy whenever yRx . Algebraically, this requirement may be



phrased: $R = R^{-1}$. At the other extreme, the relation R is anti-symmetric iff it is never the case that both xRy and yRx . In other words, R is transitive iff whenever xRy and yRz then xRz . In terms of the composition of relation, the relation R is transitive if and only if $R \bullet R \subset R$. It follows that, if R is transitive, then $R^{-1} \bullet R^{-1} = (R \bullet R)^{-1} \subset R^{-1}$ and hence the inverse of a transitive relation is transitive. If R is both transitive and reflexive, then $R \bullet R \supset R \bullet \Delta$ and hence $R \bullet R = R$; in the usual terminology, such a relation is idempotent under composition. An equivalent relation is a reflexive, symmetric, and transitive relation. Equivalence relations have a very simple structure, which we now proceed to describe. Suppose that R is an equivalent relation and that X is a domain of R . A subset A of X is an equivalent class (an R -equivalence class) if and only if there is a member x of A such that A is identical with the set of all y such that xRy . In other words, A is an equivalence class iff there is x in A such that $A = R[\{x\}]$. The fundamental result on equivalence relations states that the family a of all equivalence classes is disjoint, and that a point x is R -related to a point y if and only if both x and y belong in a class A is simply $A \times A$, which leads to the following concise formulation of the theorem.

A.3 Proof of Theorem 5

If R is an equivalence relation, then R is transitive: if yRx and zRy , then zRx . In other words, if xRy , then $R[\{y\}] \subset R[\{x\}]$. But R is symmetric (xRy whenever yRx), from which it follows that, if xRy , then $R[\{x\}] = R[\{y\}]$. If z belongs to both $R[\{x\}]$ and $R[\{y\}]$, then $R[\{x\}] = R[\{y\}]$, and consequently two equivalence classes either coincide or are disjoint. If y and z belong to the equivalence class $R[\{x\}]$, then, since $R[\{y\}] = R[\{x\}]$, it follows that yRz or, in other words, $R[\{x\}] \times R[\{x\}] \subset R$. Hence the union of $A \times A$ for all equivalence classes A is a subset of R , and since R is reflexive, if xRy , then $(x, y) \in R[\{x\}] \times R[\{x\}]$. Hence, $R = \bigcup \{A \times A : A \in a\}$. The straightforward proof of the converse is omitted.

We are frequently interested in the behavior of a relation for points belonging to a subset of its domain, and frequently the relation possesses properties for these points which it fails to have for all points. Given a set X and a relation R one may construct a new relation $R \cap (X \times X)$ whose domain is a subset of X . For convenience we will say that a relation R has a property on X , or that R restricted to X has the property iff $R \cap (X \times X)$ has the property. For example, R is transitive on X iff $R \cap (X \times X)$ is a transitive relation. This amounts to

asserting that the defining property holds for points on X ; in this case, whenever x , y , and z are points on X such that xRy and yRz , then xRz .

A.4 Proof of Theorem 7

Only part (e) will be proved. A point x is a member of $f^{-1}[\bigcap\{X_c : c \in C\}]$ if and only if $f(x)$ belongs to this intersection, which is the case iff $f(x) \in X_c$ for each c in C . but the latter condition is equivalent to $x \in f^{-1}[X_c]$ for each c in C ; that is, $x \in \bigcap\{f^{-1}[X_c] : c \in C\}$

The foregoing theorem is often summarized as: the inverse of a function preserves relative complements, unions, and intersections. It should be noted that the validity of these formulae does not depend upon the sets A and B being subsets of the range of the function. Thus, $f^{-1}[A]$ is identical with the inverse image of the intersection of A with the range of f . However, it is convenient not to restrict the notation here (and the corresponding notation for images under f) to subsets of the range (respectively, the domain). The composition of two functions is again a function by a straightforward argument. If f is a function, then $f^{-1} \bullet f$ is an equivalence relation, for $(x, y) \in f^{-1} \bullet f$ if and only if $f(x) = f(y)$. The composition $f \bullet f^{-1}$ is a function; it is the identity on the range of f .

B :Implementation of A.H.P in c++

```
/* File: AHP.cpp*/
#include <algorithm>
#include <iostream>
#include "AHP.h"
AHP::AHP(int mysize) {
    this->size = mysize;
}
AHP::AHP(constAHP&orig) {
}
AHP::~AHP() {
```

```
std::cout<< "done...";
}
void AHP::add(double *objectSet) {
    this->dataSet.push_back(objectSet);
}
double* AHP::getSumOfRoll() {
    int index = 0;
    double *rollSum = new double[this->size];
    this->DOM = this->degreeOfImportance();
    while (index < this->size) {
        double sum = 0;
        for (int i = 0; i < this->DOM.size(); i++) {
            sum += *(this->DOM.at(index) + i);
        }
        *(rollSum + index) = sum;
        index += 1;
    }return rollSum;
}
double* AHP::normalize() {
    double *trans = new double[this->size];
    double *sumRoll = this->getSumOfRoll();
    for (int i = 0; i < this->DOM.size(); i++) {
        double sum = 0;
        for (int j = 0; j < this->size; j++) {
            sum += *(DOM.at(j) + i) / *(sumRoll + j));
        }
        *(trans + i) = (sum / this->size);
    }return trans;
}
vector<double *>AHP::degreeOfImportance() {
    int index = 0;
    vector<double *> rolls;
    while (index < this->size) {
```

```
double *pRoll = new double[this->dataSet.size()];
double myRoll[this->dataSet.size()];
for (int i = 0; i < this->dataSet.size(); i++) {
myRoll[i] = *(dataSet.at(i) + index);
}
sort(myRoll, myRoll + this->size);
double least = myRoll[0];
for (int i = 0; i < this->dataSet.size(); i++) {
*(pRoll + i) = *(dataSet.at(i) + index) / least;
}
rolls.push_back(pRoll);
index += 1;
} return rolls;
}

double* AHP::resultingVector() {
double *mydata = this->normalize();
double *resVec = new double[this->size];
for (int i = 0; i < this->dataSet.size(); i++) {
double sum = 0;
for (int j = 0; j < this->size; j++) {
sum += *(dataSet.at(i) + j) * *(mydata + j));
}
*(resVec + i) = sum / *(mydata + i);
}return resVec;
}

ResultSetAHP::getResultSet() {
double *resultingVector = this->resultingVector();
double sum = 0;
for (int i = 0; i < this->size; i++) {
sum += *(resultingVector + i);
}
this->resultset.lamdaMax = sum / this->size;
this->resultset.CI = (this->resultset.lamdaMax - this->size) / (this-
```

```
>size - 1);
    this->resultset.RI = (1.98 * (this->size - 2)) / this->size;
    this->resultset.CR = this->resultset.CI / this->resultset.RI;
    return this->resultset;
}
#include <cstdlib>
#include <iostream>
#include <vector>
#include "AHP.h"
using namespace std;
int size;
int numberOfProducts;
void getProducts(AHP *ahp);
double *getValues();
int main() {
    cout<< "Specify the numbers of parameters needed: ";
    cin>> size;
    cout<< "Provide the number of products: ";
    cin>>numberOfProducts;
    AHP ahp = AHP(size);
    getProducts(&ahp);
    vector<double *> v = ahp.degreeOfImportance();
    cout<< "Degree of importance /n/n/n ";
    for (int i = 0; i<v.size(); i++){
        cout<< v.at(i);
    }
    cout<< "Degree of importance ends here /n/n/n ";
    resultSet result = ahp.getResultSet();
    cout<< "\n\nlamda max = ";
    cout<<result.lamdaMax;
    cout<< "\nC.I = ";
    cout<< result.CI;
    cout<< "\nR.I = ";
```

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```
    cout<<result.RI;
    cout<< "\nC.R = ";
    cout<< result.CR;
    cout<< "\n\n";
    return 0;
}
double *getValues() {
\n";
    cout<< "\n\n\nInput the values of your product parameters:
\n";
    double *values = new double[size];
    for (int i = 0; i< size; i++) {
    cin>> *(values + i);
    }return values;
}
void getProducts(AHP *ahp) {
    for (int i = 0; i<numberOfProducts; i++) {
    ahp->add(getValues());
    }
}
```